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Building Integration of Solar Thermal Systems – TU1205 – BISTS

ITC

Optical and thermal modelling of BISTS

Dorota Chwieduk
Institute of Heat Engineering, ITC
Faculty of Power and Aeronautical Engineering, MEiL
Warsaw University of Technology, PW

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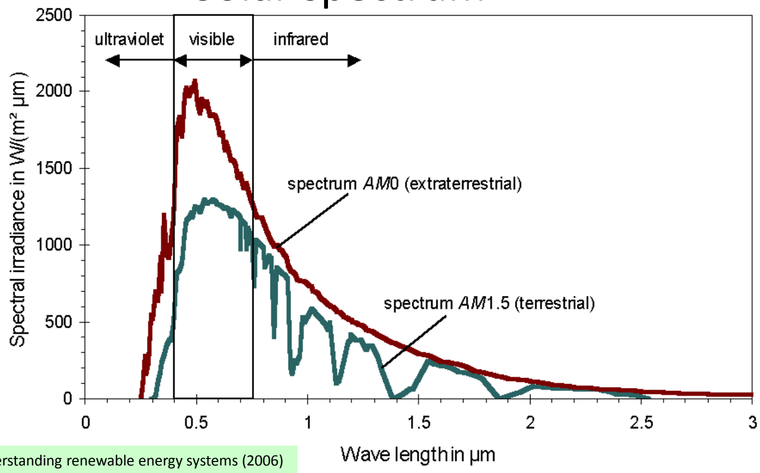
Solar spectrum



Ref. V. Quaschning Understanding renewable energy systems (2006)

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Solar energy is carried by electromagnetic waves of different lengths.

As a wave, solar radiation is a subject to the same phenomena as any other electromagnetic wave, i.e. reflection, refraction, absorption, polarization , etc.

Radiation incident on a body may be reflected by that body, absorbed within and transmitted through the body and outside.

Share of each of those processes depends on:

- **type of body (material),**
- **properties of its surface,**
- **distance travelled by radiation within the body,**
- **wavelength of the radiation,**
- **angle of incidence.**

Radiation absorption occurs at different wavelengths . Absorptance α is therefore defined as a ratio of absorbed radiation to the total incident radiation.

$$\alpha = \frac{\phi_{abs}}{\phi_{tot}}$$

Total absorptance of a surface depends on the spectrum of incident radiation

$$\alpha = \frac{\int_{\lambda=0}^{\infty} \alpha_\lambda \phi_{\lambda,tot} d\lambda}{\int_{\lambda=0}^{\infty} \phi_{\lambda,tot} d\lambda}$$

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total absorptance, reflectance and transmittance of given surface satisfy the equation:

$$\alpha + \rho + \tau = 1$$

Values of each of those three radiation properties may vary from 0 to 1.

Three perfect cases may be listed:

•white body, i.e. body which reflects all incident radiation, whose reflectance ρ = 1, while τ = 0 and α = 0;

•black body, which absorbs all incident radiation, whose absorptance α = 1, while τ = 0 and ρ = 0;

•transparent body which fully transmits incident radiation, whose transmittance τ = 1, while α = 0 and ρ = 0.

Clean air is considered to be fully transparent for solar radiation τ = 1

Transmisivity, i.e. ability to transmit radiation.

It depends on:

- **radiation angle of incidence,**
- **spectrum of radiation,**
- **polarization state of radiation.**

It is possible to distinguish :

- **directional transmisivity - in reference to radiation transmitted directionally,**
- **diffuse transmittance - referring to radiation transmitted in diffuse form.**

Total transmisivity is a sum of directional transmisivity and diffuse transmisivity, .

Transmisivity is considerably affected by harshness of a body surface.
For example for a 2 mm thick glass whose one external face is grinded, and the other polished, transmittance is 0.85 and 0.78 respectively, depending on which face is exposed to radiation.

Transparent body. Optics

Material's characteristic :

τ - transmitivity
ρ - reflectivity
α *- absorptivity*

Surface's characteristic:

τ - transmitance
ρ - reflectance
α *- absorptance*

Energy Conservation Law:

$\alpha(\lambda)+\rho(\lambda)+\tau(\lambda)=1$

Kirchoff Law

$\alpha(\lambda)=\varepsilon(\lambda)$

Transparent body:

$\tau=1,\ \rho=0,\ \alpha=0.$

Opaque body:

$\tau=0,\ \rho+\alpha=1$

Glass practically opaque for thermal radiation

$\alpha_{long\ w}=\varepsilon_{long\ w}=1-\rho_{long\ w}$

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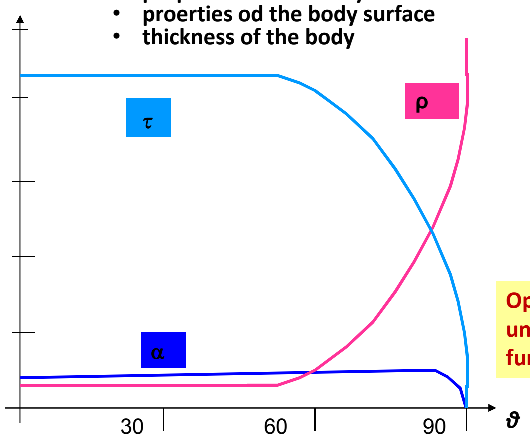
Transmitance of transparent real bodies < 1 and depends on:

- incident angle of radiation
- wave length of incident radiation ,
- properties of the body
- proerties od the body surface
- thickness of the body



$\alpha(\vartheta)+\rho(\vartheta)+\tau(\vartheta)=1$

Optical parameters of a single pane glass under incident solar radiation in a function of incident angle of the radiation

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About 99% of solar radiation spectrum is visible radiation and near-infrared radiation.

Wave absorption is conversion of wave energy into other forms of energy as a result of interaction with a medium or body, through which the wave propagates or which blocks wave propagation.

In case of light absorption, the radiation energy is converted into different kinds of internal energy or into energy of secondary radiation emitted in a different direction.

In case of near-infrared radiation, absorbed energy increases internal energy of a body, causing its temperature to rise -which may be used in solar heating systems and may be further emitted away.

Solar energy absorbed by a body, or transmitted by a transparent body and then absorbed by another body or a medium, **is converted into heat**.
Solar radiation may be reflected from certain body and may reach another body, and get absorbed there and converts into heat

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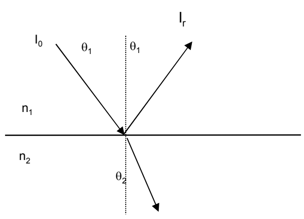
Fresnel formulated a law describing reflection nonpolarized light incident on a smooth surface and passing from a medium 1 with refractive index n_1 to medium 2 with refractive index n_2.
He derived a **relation** describing **reflection coefficients *r*,** separately addressing component with perpendicular polarization (s-polarized) r_s and that with parallel polarization (p-polarized) r_p

Fresnel Law - Reflection coefficient *r* - the ratio of reflected radiation I_r to the incident radiation I_0 on the surface.



$$r = \frac{I_r}{I_o} = \frac{1}{2}\left(r_s + r_p\right) = \frac{1}{2}\left(\frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)} + \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)}\right)$$

Snell law - relation of refractive indices of both media

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1}$$

If the radiation strikes interface between two media as **normal radiation**

$$r(0) = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

Passage of nonpolarized light beam through interface between two media

If one of the media is air $n_2 \to 1$ $\quad r(0) = \left(\frac{n_1 - 1}{n_1 + 1}\right)^2$

There are three main equivalent concepts for solving the repeated reflection problem:

- **Rubin's ray tracing method (1982),**
- Edwards's energy balance method (1977),
- Harbecke's matrix transformation method (1986).

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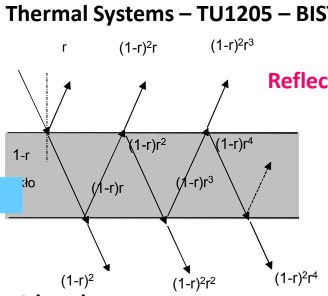
air

Transmission and repeated reflection of radiation within a transparent plate

Equation for ***transmittance*** and ***reflectance***

air



Reflection coefficient *r*

properties in reference to the s-polarized radiation

the addends of sums constitute convergent geometric series

$$\tau_s = (1-r_s)^2 + (1-r_s)^2 r_s^2 + (1-r_s)^2 r_s^4 + \ldots = (1-r_s)^2\left(1 + r_s^2 + r_s^4 + \ldots\right) = (1-r_s)^2 \sum_{n=0}^{\infty} r_s^{2n}$$

$$\tau_s = \frac{(1-r_s)^2}{1-r_s^2} = \frac{1-r_s}{1+r_s}$$

$$\tau_c = \frac{1}{2}\left(\frac{1-r_s}{1+r_s} + \frac{1-r_p}{1+r_p}\right)$$

$$\rho_s = r_s + (1-r_s)^2 r_s + (1-r_s)^2 r_s^3 + (1-r_s)^2 r_s^5 + \ldots = r_s + \left((1-r_s)^2 r_s\right)\sum_{n=0}^{\infty} r_s^{2n} -$$

$$\rho_s = r_s + \frac{\left((1-r_s)^2 r_s\right)}{1-r_s^2}$$

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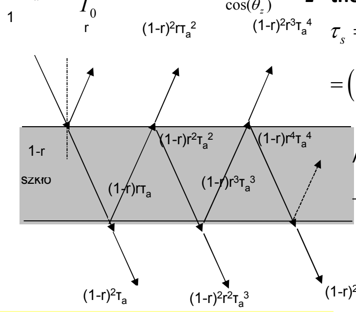
Ability to absorb radiation within a partially transparent medium is addressed by the Bouguer Law (Bouguer-Labert-Beer).

Transmission factor τ_a - ratio of radiation transmitted by the medium I_{trans}, which gets attenuated due to absorption processes within that medium, to the total incident radiation I_0

$$\tau_a = \frac{I_{trans}}{I_0} = e^{(-KL)} \qquad L = \frac{\delta}{\cos(\theta_z)}$$

L **- the length of radiation's path within transparent body**



$$\tau_s = (1-r_s)^2\tau_a + (1-r_s)^2 r_s^{\,2}\tau_a^{\,3} + (1-r_s)^2 r_s^{\,4}\tau_a^{\,5} + (1-r_s)^2 r_p^{\,6}\tau_a^{\,7} + \ldots = $$
$$= \left((1-r_s)^2\tau_a\right)\sum_{n=0}^{\infty}(r_s^{\,2n}\tau_a^{\,2n})$$

$$\tau_s = \frac{(1-r_s)^2\tau_a}{1-r_s^{\,2}\tau_a^{\,2}}$$

$$\rho_s = r_s + (1-r_s)^2 r_s\tau_a^{\,2} + (1-r_s)^2 r_s^{\,3}\tau_a^{\,4} + (1-r_s)^2 r_s^{\,5}\tau_a^{\,6} +$$
$$+(1-r_s)^2 r_s^{\,7}\tau_a^{\,8} + \ldots\ldots = r_s + \left((1-r_s)^2 r_s\tau_a^{\,2}\right)\sum_{n=0}^{\infty}(r_s^{\,2}\tau_a^{\,2})$$

$$\rho_s = r_s + \frac{(1-r_s)^2 r_s\tau_a^{\,2}}{1-r_s^{\,2}\tau_a^{\,2}}$$

$$\alpha_s = (1-r_s)(1-\tau_a) + (1-r_s)(1-\tau_a)r_s\tau_a + (1-r_s)(1-\tau_a)r_s^{\,2}\tau_a^{\,2} +$$
$$+(1-r_s)(1-\tau_a)r_s^{\,3}\tau_a^{\,3} + (1-r_s)(1-\tau_a)r_s^{\,4}\tau_a^{\,4} + \ldots\ldots =$$
$$= \left((1-r_s)(1-\tau_a)\right)\sum_{n=0}^{\infty}\left(r_p\tau_a\right)$$

$$\alpha_s = \frac{(1-r_s)(1-\tau_a)}{1-r_s\tau_a}$$

Repeated reflection in a transparent plate including absorption

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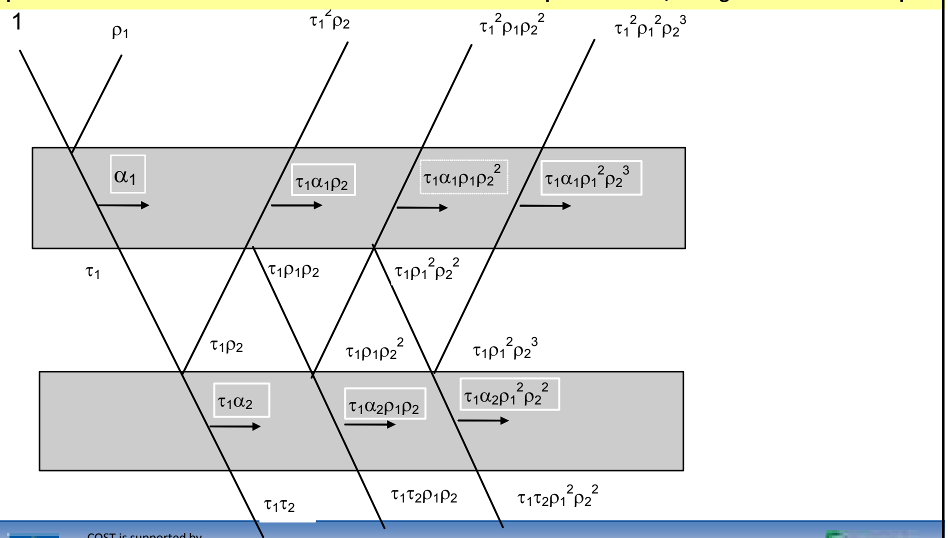
Repeated reflection at interfaces between the air and two transparent covers, taking into account absorption



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Transmitivity **and** ***reflectivity*** **and** ***absorptivity*** **of a system of two covers**

for the s-polarized radiation - **infinite sums create relevant convergent geometric series**

$$\tau_s = \tau_{s,1}\tau_{s,2} + \tau_{s,1}\tau_{s,2}\rho_{s,1}\rho_{s,2} + \tau_{s,1}\tau_{s,2}\rho_{s,1}{}^2\rho_{s,2}{}^2 + \tau_{s,1}\tau_{s,2}\rho_{s,1}{}^3\rho_{s,2}{}^3 +$$
$$+\tau_{s,1}\tau_{s,2}\rho_{s,1}{}^4\rho_{s,2}{}^4 + \ldots = \left(\tau_{s,1}\tau_{s,2}\right)\sum_{n=0}^{\infty}\left(\rho_{s,1}\rho_{s,2}\right) = \frac{\tau_{s,1}\tau_{s,2}}{1-\rho_{s,1}\rho_{s,2}}$$

$$\rho_s = \rho_{s,1} + \tau_{s,1}{}^2\rho_{s,2} + \tau_{s,1}{}^2\rho_{s,1}\rho_{s,2}{}^2 + \tau_{s,1}{}^2\rho_{s,1}{}^2\rho_{s,2}{}^3 + \tau_{s,1}{}^2\rho_{s,1}{}^3\rho_{s,2}{}^4 +$$
$$+\tau_{s,1}{}^2\rho_{s,1}{}^4\rho_{s,2}{}^5 + \ldots = \rho_{s,1} + \left(\tau_{s,1}{}^2\rho_{s,2}\right)\sum_{n=0}^{\infty}\left(\rho_{s,1}\rho_{s,2}\right) = \rho_{p,1} + \frac{\tau_{s,1}{}^2\rho_{s,2}}{1-\rho_{s,1}\rho_{s,2}}$$

$$\alpha_s = \alpha_{1,s} + \tau_{s,1}\alpha_{s,2} + \tau_{s,1}\alpha_{s,1}\rho_{s,2} + \tau_{s,1}\alpha_{s,2}\rho_{s,1}\rho_{s,2} +$$
$$+\tau_{s,1}\alpha_{s,1}\rho_{s,1}\rho_{s,2}{}^2 + \tau_{s,1}\alpha_{s,2}\rho_{s,1}{}^2\rho_{s,2}{}^2 + \tau_{s,1}\alpha_{s,1}\rho_{s,1}{}^2\rho_{s,2}{}^3 +$$
$$+\tau_{s,1}\alpha_{s,2}\rho_{s,1}{}^3\rho_{s,2}{}^3 + \tau_{s,1}\alpha_{s,1}\rho_{s,1}{}^3\rho_{s,2}{}^4 + \ldots =$$
$$= \alpha_{1,s} + \tau_{s,1}(\alpha_{s,2} + \alpha_{s,1}\rho_{s,2}) + \tau_{s,1}(\alpha_{s,2} + \alpha_{s,1}\rho_{s,2})\rho_{s,1}\rho_{s,2} +$$
$$+\tau_{s,1}(\alpha_{s,2} + \alpha_{s,1}\rho_{s,2})\rho_{s,1}{}^2\rho_{s,2}{}^2 + \tau_{s,1}(\alpha_{s,2} + \alpha_{s,1}\rho_{s,2})\rho_{s,1}{}^3\rho_{s,2}{}^3 + \ldots =$$
$$= \alpha_{1,s} + \frac{\tau_{s,1}(\alpha_{s,2} + \alpha_{s,1}\rho_{s,2})}{1-\rho_{s,1}\rho_{s,2}}$$

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For the general case of *n* covers for the p-polarized component

$$\tau_p = \frac{\tau_{p,n-1}\tau_{p,n}}{1-\rho_{p,n-1}\rho_{p,n}} \qquad \rho_p = \rho_{p,n-1} + \frac{\tau_{p,n-1}{}^2\rho_{p,n}}{1-\rho_{p,n-1}\rho_{p,n}} \qquad \alpha_p = \alpha_{p,n-1} + \frac{\tau_{p,n-1}(\alpha_{p,n} + \alpha_{p,n-1}\rho_{p,n})}{1-\rho_{p,n-1}\rho_{p,n}}$$

After appropriate transformations for the polarized components

$$\tau_p = \frac{(1-r_p)^2\tau_a}{1-r_p{}^2\tau_a{}^2} = \tau_a\left(\frac{1-r_p}{1+r_p}\right)\left(\frac{1-r_p{}^2}{1-r_p{}^2\tau_a{}^2}\right) \qquad \tau_s = \frac{(1-r_s)^2\tau_a}{1-r_s{}^2\tau_a{}^2} = \tau_a\left(\frac{1-r_s}{1+r_s}\right)\left(\frac{1-r_s{}^2}{1-r_s{}^2\tau_a{}^2}\right)$$

The last addend in practice is very close to 1

$$\tau = \tau_a\frac{1}{2}\left(\left(\frac{1-r_p}{1+r_p}\right) + \left(\frac{1-r_s}{1+r_s}\right)\right) \longrightarrow \tau = \tau_c\tau_a$$

For *n* parallel and identical covers total transmittance τ_n

$$\tau_n = \frac{1}{2}\left(\frac{1-r_p}{1+(2n-1)r_p} + \frac{1-r_s}{1+(2n-1)r_s}\right)e^{(-K\sigma/\cos(\theta_2))}$$

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simplifications for analyzing properties of transparent covers

$$\alpha \cong 1 - \tau_a \qquad \rho \cong \tau_a - \tau_c = \tau_a (1 - \tau_\rho \tau_s)$$

In case of two non-identical transparent covers

$$\tau = \frac{1}{2}\left[\left(\frac{\tau_1 \tau_2}{1-\rho_1\rho_2}\right)_p + \left(\frac{\tau_1 \tau_2}{1-\rho_1\rho_2}\right)_r\right] \cong \left(\frac{\tau_1 \tau_2}{1-\rho_1\rho_2}\right)$$

$$\rho = \frac{1}{2}\left[\left(\rho_1 + \frac{\rho_2 \tau_1^{\,2}}{1-\rho_1\rho_2}\right)_p + \left(\rho_1 + \frac{\rho_2 \tau_1^{\,2}}{1-\rho_1\rho_2}\right)_r\right] \cong \left(\rho_1 + \frac{\rho_2 \tau_1^{\,2}}{1-\rho_1\rho_2}\right)$$

$$\alpha = \frac{1}{2}\left[\left(\alpha_1 + \frac{\tau_1(\alpha_2 + \alpha_1\rho_2)}{1-\rho_1\rho_2}\right)_p + \left(\alpha_1 + \frac{\tau_1(\alpha_2 + \alpha_1\rho_2)}{1-\rho_1\rho_2}\right)_r\right] \cong \left(\alpha_1 + \frac{\tau_1(\alpha_2 + \alpha_1\rho_2)}{1-\rho_1\rho_2}\right)$$

identical transparent covers

$$\tau_c = \left(\frac{\tau^2}{(1-\rho^2)}\right)$$

$$\rho_c = \left(\rho + \frac{\rho\tau^2}{1-\rho^2}\right)$$

$$\alpha_c = \left(\alpha + \frac{\tau\alpha\ (1+\rho\ )}{(1-\rho^2)}\right)$$

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In case of two non-identical transparent covers - for the outer one

$$\alpha_{s,out} = \alpha_{1,s} + \tau_{s,1}\alpha_{s,1}\rho_{s,2} + \tau_{s,1}\alpha_{s,1}\rho_{s,2}{}^2\rho_{s,1} + \tau_{s,1}\alpha_{s,1}\rho_{s,2}{}^3\rho_{s,2}{}^2 + \dots$$

$$= \alpha_{1,s} + \tau_{s,1}\alpha_{s,1}\rho_{s,2}\sum_{n=0}^{\infty}\left(\rho_{s,2}\cdot\rho_{s,1}\right) = \alpha_{1,s} + \frac{\tau_{s,1}\alpha_{s,1}\rho_{s,2}}{1-\rho_{s,1}\rho_{s,2}} = \alpha_{1,p}\left(1 + \frac{\tau_{s,1}\rho_{s,2}}{1-\rho_{s,1}\rho_{s,2}}\right)$$

for the inner one

$$\alpha_{s,in} = \tau_{s,1}\alpha_{s,2} + \tau_{s,1}\alpha_{s,2}\rho_{s,1}\rho_{s,2} + \tau_{s,1}\alpha_{s,2}\rho_{s,1}{}^2\rho_{s,2}{}^2 + \dots$$

$$= \tau_{s,1}\alpha_{s,2}\sum_{n=0}^{\infty}\left(\rho_{s,1}\cdot\rho_{s,2}\right) = \frac{\tau_{s,1}\alpha_{s,2}}{1-\rho_{s,1}\rho_{s,2}} = \alpha_{2,s}\frac{\tau_{s,1}}{1-\rho_{s,1}\rho_{s,2}}$$

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optical properties for identical covers

$\alpha_1 = \alpha_2 = \alpha$; $\tau_1 = \tau_2 = \tau$; $\rho_1 = \rho_2 = \rho$

$$\alpha_{out} = \alpha\left(1 + \frac{\tau \cdot \rho}{1 - \rho^2}\right)$$

$$\alpha_{in} = \alpha\left(\frac{\tau}{1 - \rho^2}\right)$$

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Key parameters of glass used to make covers

Characteristic parameter	Common glass (pane)	Glass with lowered iron content	Low-iron glass (high-transparency glass)
Iron oxide content [%]	0.12	0.05	0.01
Refractive index	1.52	1.50	1.50
τ (along normal) [%]	79 -84	88-89	91-92
Thickness [cm]	0.32 - 0.64	0.32 - 0.48	0.32 - 0.56

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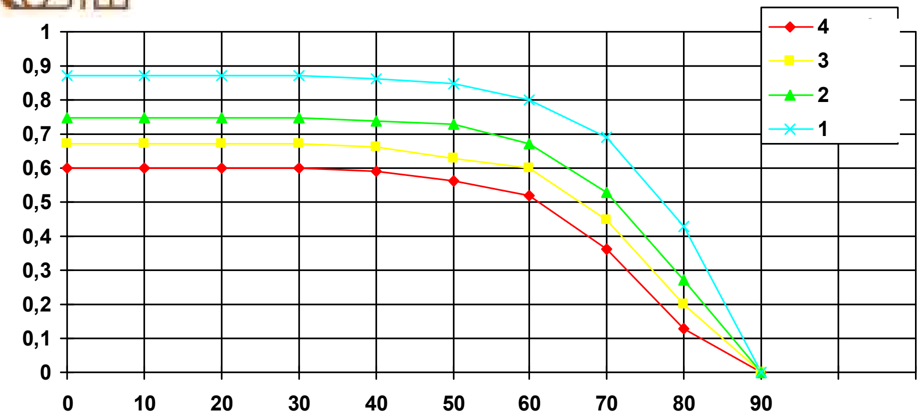

Changes of transmisivity of a glass pane system composed of different number of panes

Example based on data from Duffie J. A., Beckman W. A.: *Solar Engineering of Thermal Processes*, John Wiley & Sons, Inc., New York, 1991, pp. 225

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According to Brandemuehl and Beckman **the equivalent angle of incidence of diffuse radiation on a tilted surface,** for which transmittance is determined using isotropic model, varies from **55° to 60°**
Equivalent angle of incidence of diffuse radiation on a tilted surface:

$$\theta_d = 59.68° - 0.1388\beta + 0.0011497\beta^2$$

The equivalent angle of incidence for radiation reflected from surroundings incident on a tilted surface used to determine transmittance varies from **60° to 90°**:

$$\theta_r = 90° - 0.5788\beta + 0.002693\beta^2$$

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Phenomena of repeated reflection between absorbing surface and its cover are mathematically described by equation for total transmittance-absorptance product

$$(\tau\alpha) = \tau\alpha\sum_{n=1}^{\infty}[(1-\alpha)\rho_d]^n = \frac{\tau\alpha}{1-(1-\alpha)\rho_d}$$

Solar radiation penetration inside cavity - the concept of effective absorptance is applied

$$\alpha_{eff} = \frac{\alpha_i}{\alpha_i + (1-\alpha_i)\frac{A_a}{A_i}}$$

α_i – absorptance of surface of an internal absorbing element,
A_a – aperture area of transparent surface (glazing), [m^2].
A_i – total area of internal absorbing surfaces, [m^2].

Share of solar energy, which passes through glazing and is absorbed by interior is described by the effective transmittance-absorptance product

$$\tau_c\alpha_{eff} = \tau_c\frac{\alpha_i}{\alpha_i + (1-\alpha_i)\tau_d\frac{A_a}{A_i}}$$

Cover-absorber system, solar energy is absorbed by a tilted surface

$$I_{sa} = I_b R_b(\tau\alpha)_b + I_d(\tau\alpha)_d\left(\frac{1+\cos\beta}{2}\right) + \rho_g(I_b + I_d)(\tau\alpha)_g\left(\frac{1-\cos\beta}{2}\right)$$

Value $(\tau\alpha)_d$ can be assumed to be constant, with a value equal to **$(\tau\alpha)_b$ for beam radiation incident at 60°** or for **the equivalent angle of incidence of diffuse radiation** .

In a special case of a **vertical absorber with a transparent cover** analyzed with isotropic solar diffuse radiation model: **$(\tau\alpha)_d = (\tau\alpha)_g = (\tau\alpha)_{b60}$**.

$$I_{sa}(t) = I_b(t)R_b(t)(\tau\alpha)_b + (\tau\alpha)_{b60}\{0.5[I_d(t) + \rho_g(I_b(t) + I_d(t))]\}$$

From optics to heat transfer

Solar radiation is coming through transparent cover and is absorbed by opaque body

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Solar radiation is coming through transparent cover and is absorbed by opaque body

$$P_{cover} = A_c G_t$$

$$P_{absorber} = A_c G_t \tau$$

$$P_{absorbed} = A_c G_t \tau\alpha \approx A_c G_t \eta_o$$

G_t = **Irradiance on the tilted surface (W/m²)**

τ = **Transmissivity of the cover**

α = **Absorbtivity at solar wavelengths**

A_c = **Collector area (m²)**

η_o = **Optical efficiency**

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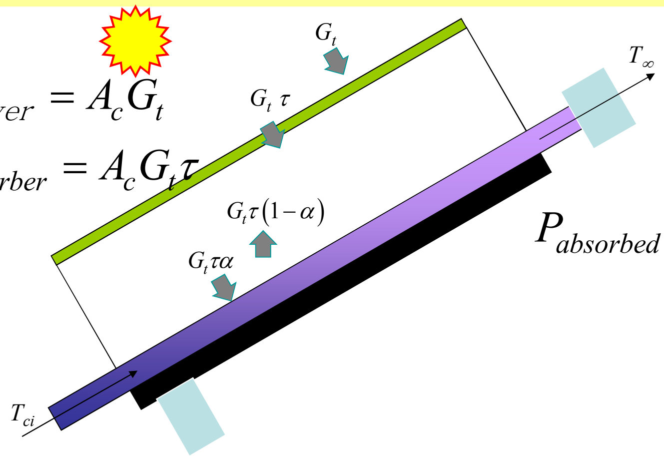
Solar radiation is coming through transparent cover and is absorbed by opaque body

$$P_{cover} = A_c G_t$$

$$P_{absorber} = A_c G_t \tau$$



$$P_{absorbed} = A_c G_t \tau\alpha \approx A_c G_t \eta_o$$

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Hottel-Whillier-Bliss equation

$$Q = A_c F_R \left[G_t \eta_o - U_L \left(T_{ci} - T_A \right) \right]$$

- A = Collecting area (m²)
- F_R = Heat removal factor - (Overall heat transfer characteristics of collecting body - ratio of the actual amount of heat transferred to the heat gaining fluid, to the heat which would be transferred if the entire collecting body (plate) was at the fluid inlet temperature)
- G_t = Total incident global solar iradiation (W/m²)
- η_o = The transmission and absorption are angle dependent. As we are only considering a short period either side of solar noon we use the effective η_o.
- U_L = Heat transfer (loss) coefficient from collecting body (depends on levels of insulation behind back and sides of absorber)
- T_a = Ambient temperature.
- T_{ci} = Collector fluid inlet temperature (efficiency rises with decreasing T_i, and falls with increasing outlet temperature)

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The general energy balance

Q_{in} - heat rate input, W

Q_{out} – heat rate output, W

Q_{loss} – heat losses, W

$$Vc\ \rho \frac{dT}{dt} = Q_{in}(t) - Q_{out}(t) - Q_{loss}(t)$$

$$Q = A_c F_R \left[G_t \eta_o - U_L \left(T_{ci} - T_A \right) \right]$$

$$Q_u(t) = A_c F'(t)[(\tau\alpha) G_s(t) - U_L(\bar{T}_f(t) - T_a(t))]$$

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Energy balance

$$Vc_p\rho \frac{dT_{in}}{dt} = \sum_i (\tau\alpha)_i(t) G_i(t) A_i(t) - \sum Q_{use}(t) \pm \sum Q_{loss/gain}(t)$$

$$Vc_p\rho \frac{dT_{in}}{dt} = \tau\alpha G(t) A_w - \frac{A\left(T_{in}(t) - T_{out}(t)\right)}{R(t)}$$

Tin=const $\Rightarrow$ $\frac{dT_{in}}{dt} = 0$ $\quad \tau\alpha G(t) A_w = \frac{A\left(T_{in} - T_{out}(t)\right)}{R(t)}$

$$\frac{dT_{in}}{dt} = -\frac{A\left(T_{in}(t) - T_z(t)\right)}{RC} \longrightarrow T_{in}(t) = T_{out} + (T_{in}|_{t=0} - T_{out}) e^{\left[-\frac{t}{ARC}\right]}$$

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Energy transfer through glazing

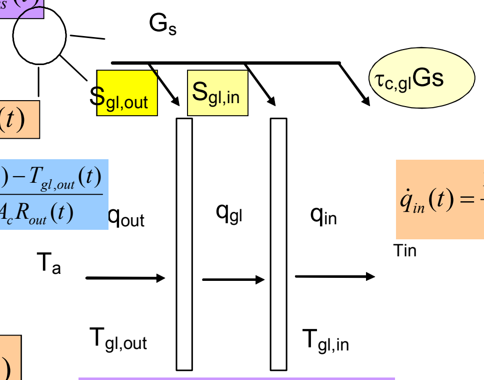
solar radiation

$\dot{q}_{out}(t) + S_{gl,out}(t) = \dot{q}_{gl,panes}(t)$

$\dot{q}_{gl,panes}(t) + S_{gl,in}(t) = \dot{q}_{in}(t)$

$$\dot{q}_{out}(t) = \frac{T_a(t) - T_{gl,out}(t)}{A_c R_{out}(t)}$$

$$\dot{q}_{in}(t) = \frac{T_{gl,in}(t) - T_{in}(t)}{A_c R_{in}(t)}$$



no solar radiation

$\dot{q}_{out}(t) = \dot{q}_{gl,panes}(t) = \dot{q}_{in}(t)$

$$\dot{q}_{gl,panes}(t) = \frac{T_{out,in}(t) - T_{gl,in}(t)}{A_c R_{gl,panes}(t)}$$

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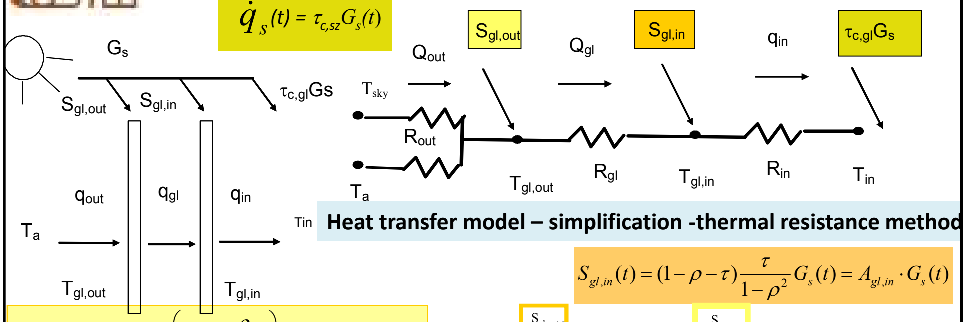
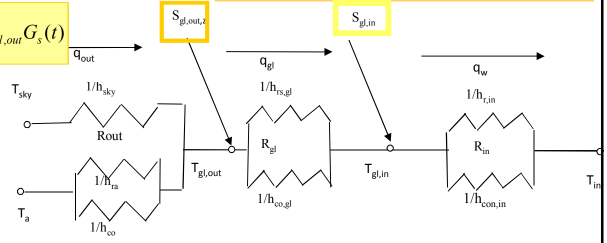
$\dot{q}_s(t) = \tau_{c,sz} G_s(t)$



Heat transfer model – simplification -thermal resistance method

$$S_{gl,in}(t) = (1-\rho-\tau)\frac{\tau}{1-\rho^2} G_s(t) = A_{gl,in} \cdot G_s(t)$$

$$S_{gl,out}(t) = (1-\rho-\tau)\left(1+\tau\frac{\rho}{1-\rho^2}\right) G_s(t) = A_{gl,out} G_s(t)$$



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Heat transfer model: glass – ambient

$$\frac{1}{A_c R_{out}(t)} = h_{con,a}(t) + \frac{\varepsilon_o \sigma (T_a(t)^4 - T_{gl,out}(t)^4)}{(T_a(t) - T_{gl,out}(t))}$$

$$\dot{q}_{out}(t) = \frac{T_a(t) - T_{gl,out}(t)}{A_c R_{out}(t)}$$

$$\frac{1}{A_c R_{gl}(t)} = h_{con,gl}(t) + \frac{\sigma}{\frac{1}{\varepsilon_o} + \frac{1}{\varepsilon_o} - 1} \frac{(T_{gl,out}(t)^4 - T_{gl,in}(t)^4)}{(T_{gl,out}(t) - T_{gl,in}(t))}$$

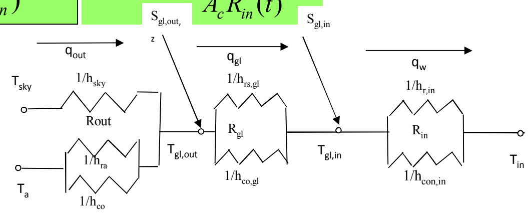
$$\dot{q}_{gl,panes}(t) = \frac{T_{out,in}(t) - T_{gl,in}(t)}{A_c R_{gl,panes}(t)}$$

$$\frac{1}{A_c R_{in}(t)} = h_{con,in}(t) + \frac{\varepsilon_o \sigma (T_{gl,in}(t)^4 - T_{in}(t)^4)}{(T_{gl,in}(t) - T_{in})}$$

$$\dot{q}_{in}(t) = \frac{T_{gl,in}(t) - T_{in}(t)}{A_c R_{in}(t)}$$

$$\dot{q}_{out}(t) + S_{gl,out}(t) = \dot{q}_{gl,panes}(t)$$

$$\dot{q}_{gl,panes}(t) + S_{gl,in}(t) = \dot{q}_{in}(t)$$



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$$\dot{q}_{gl,panes}(t) + S_{gl,in}(t) = \dot{q}_{in}(t)$$

$$h_{con,in}(T_{in}, T_{gl,in}(t))(T_{gl,in}(t) - T_{in}) + \varepsilon_o \sigma (T_{gl,in}(t)^4 - T_{in}(t)^4) =$$

$$= h_{con,gl}(T_{gl,out}(t), T_{gl,in}(t))(T_{gl,out}(t) - T_{gl,in}(t)) + \frac{\sigma}{\frac{1}{\varepsilon_o} + \frac{1}{\varepsilon_o} - 1}(T_{gl,in}(t)^4 - T_{gl,out}(t)^4) + S_{gl,in}(t)$$

$$\dot{q}_{out}(t) + S_{gl,out}(t) = \dot{q}_{gl,panes}(t)$$

$$h_{con,out}(T_a(t), T_{gl,out}(t))(T_a(t) - T_{gl,out}(t)) + \varepsilon_o \sigma (T_a(t)^4 - T_{gl,out}(t)^4) =$$

$$= h_{con,gl,out}(T_{gl,out}(t), T_{gl,in}(t))(T_{gl,out}(t) - T_{gl,in}(t)) + \frac{\sigma}{\frac{1}{\varepsilon_o} + \frac{1}{\varepsilon_o} - 1}(T_{gl,in}(t)^4 - T_{gl,out}(t)^4) - S_{gl,out,z}(t)$$

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Heat transfer coefficient for air cavity between glass panes

$$\dot{q}_{gl,panes}(t) = \frac{T_{out,in}(t) - T_{gl,in}(t)}{A_c R_{gl,panes}(t)}$$

$$\frac{1}{A_c R_{gl}(t)} = h_{con,gl}(t) + \frac{\sigma}{\frac{1}{\varepsilon_o} + \frac{1}{\varepsilon_o} - 1} \frac{(T_{gl,out}(t)^4 - T_{gl,in}(t)^4)}{(T_{gl,out}(t) - T_{gl,in}(t))}$$

For vertical air gap – Shewen formula

$$h_{con,gl,gap}(t) = Nu(t)\frac{\lambda_{gl,gap}}{L}$$

$$Nu = \left[1 + \left(\frac{0{,}0665 Ra^{1/3}}{1 + (9000/Ra)^{1/4}}\right)^2\right]^{1/2}$$

$$Ra(t) = \frac{g\beta\rho^2 c_p L^3}{\lambda\mu}\Delta T(t)$$

Wright ,when **ratio of an air gap hight to the width >20.**

$Nu_{1,90}$= 0,06738 $Ra^{1/3}$ dla $5 \odot 10^4 < Ra$

$Nu_{1,90}$= 0,02815 $Ra^{0,4134}$ dla $10^4 < Ra \leq 5 \odot 10^4$

$Nu_{1.90}$= 1+1,75967 $\odot 10^{-10}$ $Ra^{2,2985}$ dla $Ra \leq 10^4$

$$Nu_{2,90} = 0{,}242\left(\frac{Ra}{\left(\frac{H}{L}\right)}\right)^{0{,}272}$$

$Nu_{90}=(Nu_{1,90}, Nu_{2,90})_{max}$

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Heat transfer coefficient for air cavity between glass panes

For air cavity inclined β < 60⁰ – Hollands formula

For heat transfer from inside to outside

$$Nu = 1 + 1{,}446\left(1 - \frac{1708}{Ra \cdot \cos\beta}\right)^+ \left(1 - \frac{(\sin(1{,}8\beta))^{1.6} 1708}{Ra \cdot \cos\beta}\right) + \left(\left(\frac{Ra \cdot \cos\beta}{5830}\right)^{1/3} - 1\right)^+$$

For heat transfer from outside to inside

$\beta = (180^0 - \beta)$

$$Ra(t) = \frac{g\beta\rho^2 c_p L^3}{\lambda\mu}\Delta T(t)$$

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Heat transfer coefficient for air cavity between glass panes

For air cavity inclined $60^0 \leq \beta < 90^0$ – El Sherbiny

$$Ra(t) = \frac{g\beta\rho^2 c_p L^3}{\lambda\mu}\Delta T(t)$$

For heat transfer from inside to outside for **60^0**

$$G = \frac{0{,}5}{\left[1+\left(\frac{Ra}{3160}\right)^{20{,}6}\right]^{0{,}1}}$$

$$Nu_{1,60} = \left[1+\left(\frac{0{,}093Ra^{0{,}314}}{1+G}\right)^7\right]^{1/7}$$

$$Nu_{2,60} = \left(0{,}104+\frac{0{,}175}{\frac{H}{L}}\right)Ra^{0{,}283}$$

$Nu_{60}=(Nu_{1,60}, Nu_{2,60})_{max}$

$$h_{con,60} = Nu_{60}\frac{\lambda_{cav}}{L_{cav}}$$

For **$60^0 \leq \beta < 90^0$** - direct linear interpolation

$$h_{con,\beta} = h_{con,90} + \left(h_{con,60} - h_{con,90}\right)\cdot\left(\frac{90-\beta}{30}\right)$$

For $10^2 < Ra < 2 \cdot 10^7$ and $5 < (H/L) < 100$

For heat transfer from outside to inside for $\beta < 90^0$ - *Arnold formula*

$$Nu_\beta = 1 + (Nu_{90} - 1)\sin\beta$$

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Heat transfer coefficients from outside

Heat exchange through convection occurs between the face of the receiver, i.e. transparent cover or absorbing surface directly (if no cover used) and ambient air.

Convection forced by the wind:

$h_{con,out} = a + b\,u$ a = 5,7 W/(m²K); b = 3,8 [W/(m²K)]/(m/s)

a,b – ref. Mac-Adamson

$h_{con,out}$ = 5.8 W (m² K)$^{-1}$ for u = 1 [m/s] or

$h_{con,out}$ = 8.8 W (m² K)$^{-1}$ for u = 2 [m/s]

ref. Morrison

Mitchel Formula for for entire building subjected to influence of wind

$Nu = 0.42\, Re^{0.6}$ *$Re = uL / \mu$*

Thermal radiation: outer surface – ambient

$$\dot{Q}_r = A_L h_r F_a\left(T_p - T_a\right) = A_L\varepsilon\sigma\left(T_p^4 - T_a^4\right)$$

$$\rightarrow h_r = \frac{A_L\varepsilon\sigma\left(T_p^4 - T_a^4\right)}{A_L F_a\left(T_p - T_a\right)}$$

σ – Stefan–Boltzmann constant: σ = 5,67 10^{-8} W/(m² K⁴)

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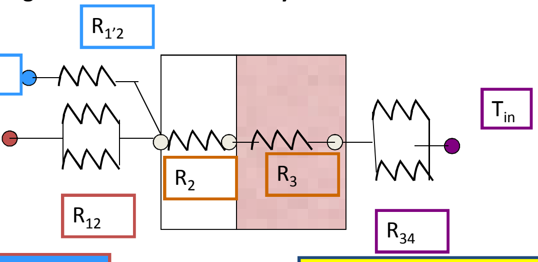
R [m² K/W]

$R_{1'2} = R_{ra,sky}$



$$\frac{1}{R_{12}} = \frac{1}{R_{12con}} + \frac{1}{R_{12ra}}$$

$$\frac{1}{R_z} = \frac{1}{R_{12}} + \frac{1}{R_{1'2}} = \frac{R_{1'2} + R_{12}}{R_{12} R_{R_{1'2}}}$$

$R_{23} = R_2 + R_3$

$$\frac{1}{R_w} = \frac{1}{R_{34con}} + \frac{1}{R_{34ra}} = \frac{R_{34ra} + R_{34con}}{R_{34con} R_{34ra}}$$

$$R_L = \left(\frac{1}{R_{12}} + \frac{1}{R_{1'2}}\right)^{-1} + R_{23} + \left(\frac{1}{R_{34}}\right)^{-1} = \frac{R_{12} R_{1'2}}{R_{1'2} + R_{12}} + R_{23} + R_{34}$$

Thermal resistance method – stable state

$$U = \frac{1}{R_L}$$

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HEAT TRANSFER THROUGH A BUILDING ENEVELOPE unsteady state

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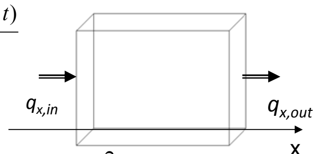
Temperature field in an opaque element of a wall

$$\frac{\partial T_{pi}(x,t)}{\partial t} = a_{pi} \frac{\partial^2 T_{pi}(x,t)}{\partial x_{pi}^{\ 2}}$$

Referring to the thermal capacity $Mc_p = V\rho c_p$ [J/K]

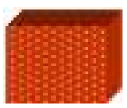
$$\frac{1}{V}(Mc_{pi})\frac{\partial T_{pi}(x,t)}{\partial t} = \lambda_{pi} \frac{\partial^2 T_{pi}(x,t)}{\partial x_{pi}^{\ 2}}$$



$$q_x = -\lambda \frac{\partial t}{\partial x} \qquad q_x + \frac{\partial q_x}{\partial x} dx$$

Building materials

High thermal capacity



High thermal insulation

$V\rho c_p$ [J/K] ↗ λ [W/mK] ↘ δ [m] ↗

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