# Basic Solar Geometry

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# Contents



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# Introduction

- The sun is a sphere of intensely hot gaseous matter with a diameter of $1.39 \times 10^9$ m.
- The solar energy strikes our planet a mere 8 minutes and 20 seconds after leaving the giant furnace, the sun which is $1.5 \times 10^{11}$m away.
- The sun's total energy output is $3.8 \times 10^{20}$ MW which is equal to 63 MW/$m^2$ of the sun's surface.
- This energy radiates outwards in all directions.
- Only a tiny fraction, $1.7 \times 10^{14}$ kW, of the total radiation emitted is intercepted by the earth.
- 30 minutes of solar radiation falling on earth is equal to the world energy demand for one year.



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# Initial applications

- Man realised that a good use of solar energy is in his benefit, from the prehistoric times.
- Since prehistory, the sun has dried and preserved man's food. It has also evaporated sea water to yield salt.
- Since man began to reason, he has recognised the sun as a motive power behind every natural phenomenon.
- This is why many of the pre-historic tribes considered Sun as "God".

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# Solar energy as a resource

- Basically all the forms of energy in the world as we know it are solar in origin.
- Oil, coal, natural gas and woods were originally produced by photosynthetic processes, followed by complex chemical reactions in which decaying vegetation was subjected to very high temperatures and pressures over a long period of time.
- Even the wind and tide energy have a solar origin since they are caused by differences in temperature in various regions of the earth.



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# Applications

- Solar energy is used to:
  - heat and cool buildings (both active and passive),
  - heat water for domestic and industrial uses,
  - heat swimming pools,
  - power refrigerators,
  - operate engines and pumps,
  - desalinate water for drinking purposes,
  - generate electricity,
  - for chemistry applications, and
  - many more.

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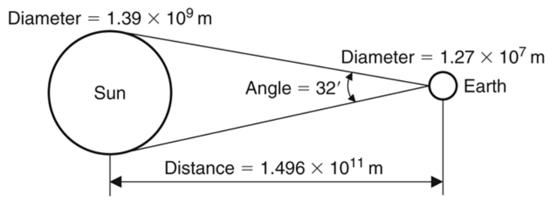
# Relation between the Earth and the Sun

- It is very important to be able to know the exact position of the sun on the sky.
- As look from the earth the sun rotates around its axis once every four weeks.



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# RECKONING OF TIME

- In solar energy calculations, apparent solar time (AST) must be used to express the time of the day.
- Apparent solar time is based on the apparent angular motion of the sun across the sky.
- The time when the sun crosses the meridian of the observer is the local solar noon. This is the highest point the sun can get in a day.
- It usually does not coincide with the clock time of the locality.
- It is necessary to convert local standard time (LST) to apparent solar time (AST) by applying two corrections:
  - Equation of time
  - Longitude correction

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# Equation of Time

- The earth's orbital velocity varies throughout the year so the apparent solar time varies slightly from the mean time kept by a clock running at a uniform rate.
- The variation is called the equation of time (ET).
- The values of the equation of time as a function of the day of the year (N) can be obtained approximately from the following equation:

$$ET=9.87 \sin(2B)-7.53 \cos(B)-1.5 \sin(B) \text{ [min]}$$

where: $B = 360 / 364 (N - 81)$

- A graphical representation of above Equation is shown in next figure from which the Equation of Time can be obtained directly.

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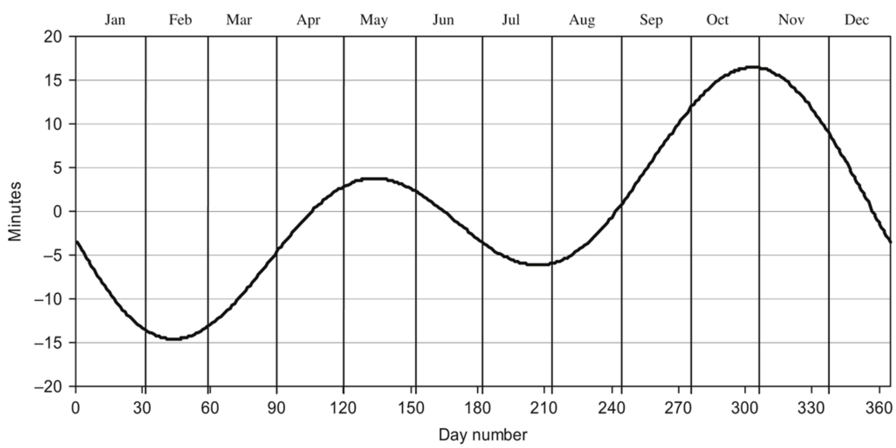
# Equation of Time graph

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# Longitude Correction

- The standard clock time is reckoned from a selected meridian near the center of a time zone or from the standard meridian, the Greenwich, which is at longitude of zero degrees.
- Since the sun takes four minutes to transverse one degree of longitude, a longitude correction term of:

  **4x(Standard Longitude - Local Longitude)**

  should be either added or subtracted to the standard clock time of the locality.
- This correction is constant for a particular longitude and the following rule must be followed with respect to the sign convention.
- If the location is east of the standard meridian, the correction is added to the clock time. If the location is west is subtracted.





# Apparent Solar Time

- For the solar energy calculations Apparent Solar Time (AST) to specify the time of day must be used.
- It is based on the relative motion of the sun in the sky.
- The time when sun passes through meridian is 12.00 (solar noon).
- At this time the sun faces exactly south.

# Apparent Solar Time

- The general equation for calculating the apparent solar time (AST) is:
  **AST = LST+ET±4 (SL - LL)**
  where: LST = Local Standard Time.
  ET = Equation of Time.
  SL = Standard Longitude.
  LL = Local Longitude.
- If a location is east of Greenwich the sign is minus (-) and if it is west the sign is plus (+).
- If a daylight saving time is used this must be subtracted from the local standard time.
- For the locality of Germany the standard longitude (SL) is 15°E. If we consider a city which is at a local longitude (LL) of 17.3° east of Greenwich, the correction is: -4*(15-17.3) = +9.2 min.
- Therefore AST equation can be written as: AST = LST+ET+9.2 [min]



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# Solar angles

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# SOLAR ANGLES

- The sun's position in the sky changes from day to day and from hour to hour.
- It is common knowledge that the sun is higher in the sky in the summer than in winter. The relative motions of the sun and earth are not simple, but they are systematic and thus predictable.
- Once a year the earth moves around the sun in an orbit that is elliptical in shape.
- As the earth makes its yearly revolution around the sun it rotates every 24 hours about its axis which is tilted at an angle of 23 degrees 27.14 min (23.45$^o$) to the plane of the elliptic which contains the earth's orbital plane and the sun's equator.

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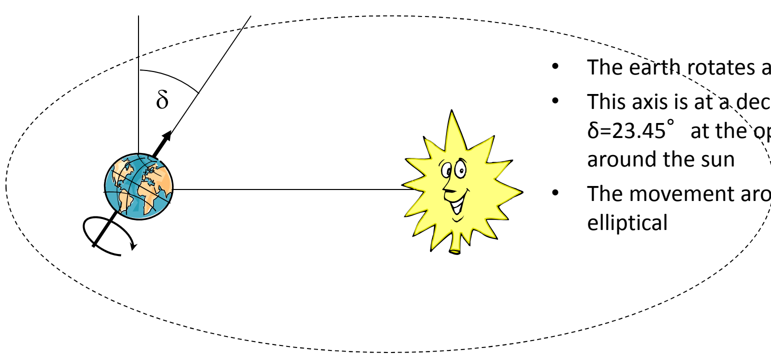
# Solar Geometry




- The earth rotates around its axis
- This axis is at a declination angle δ=23.45° at the optical plane around the sun
- The movement around the sun is elliptical

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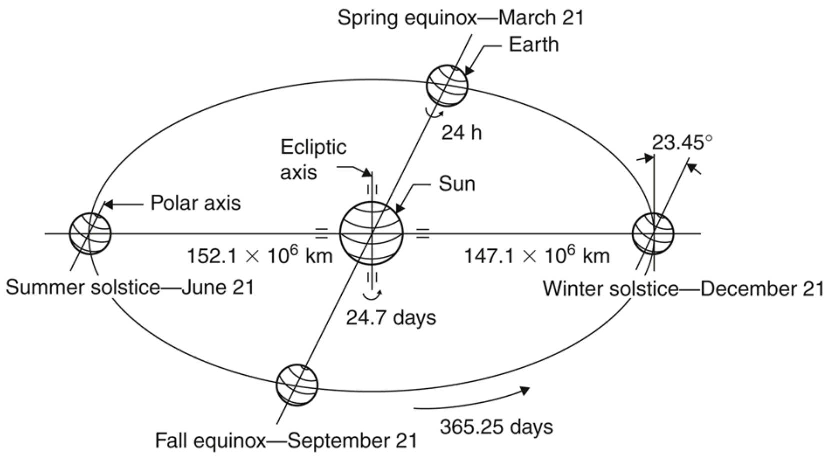
# Annual motion of the earth about the sun



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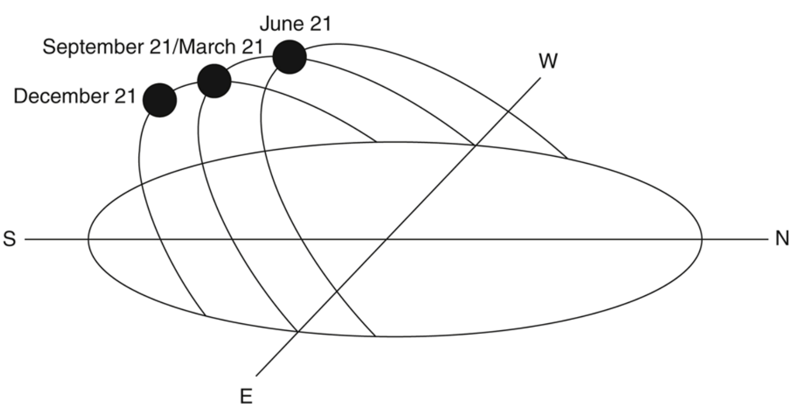
# Annual changes in the sun's position in the sky (northern hemisphere)

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# Ptolemaic view of sun motion

- Usually, the Ptolemaic view of the sun's motion is be used to the analysis of the sun angles for simplicity.
  - Since all motion is relative, it is convenient to consider the earth fixed and to describe the sun's virtual motion in a coordinate system fixed to the earth with its origin at the site of interest.
- For most solar energy applications one needs reasonably accurate predictions of where the sun will be in the sky at a given time of day and year.
- In the Ptolemaic sense, the sun is constrained to move with two degrees of freedom on the celestial sphere, therefore its position with respect to an observer on earth can be fully described by means of two astronomical angles,
  - the solar altitude (α) and
  - the solar azimuth (z).



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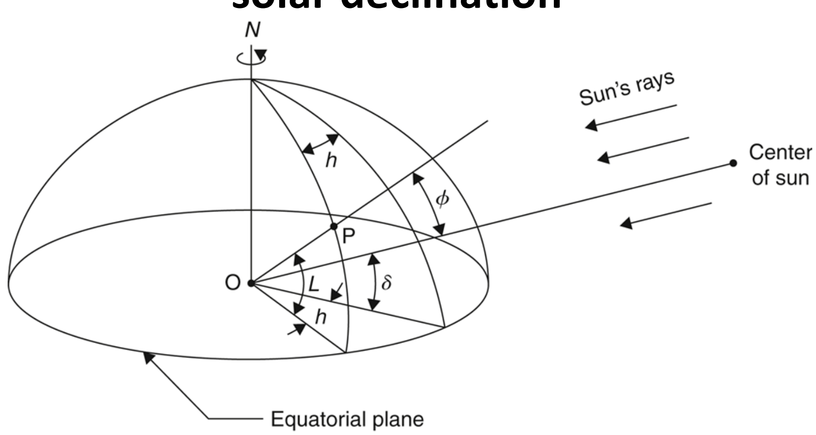
# Definition of latitude, hour angle and solar declination

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# *Declination, δ*

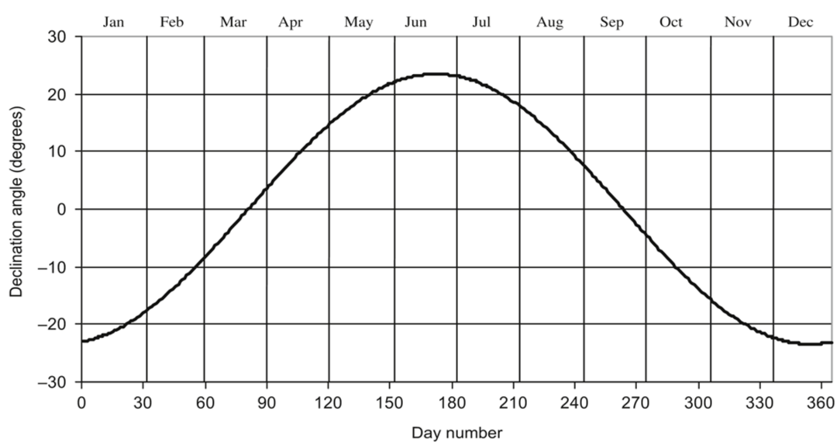
- The solar declination is the angular distance of the sun's rays north (or south) of the equator, north declination designated as positive.
- It is the angle between the sun - earth center line and the projection of this line on the equatorial plane.
- The declination angle ranges from $0^o$ at the spring equinox, to $+23.45^o$ at the summer solstice, to $0^o$ at the fall equinox, to $-23.45^o$ at the winter solstice.
- The variation of the solar declination angle throughout the year is shown next figure.
- The declination angle δ, in degrees, for any day of the year (N) can be calculated approximately by the equation:

**$\delta = 23.45 \sin[360 / 365 (284+N)]$**

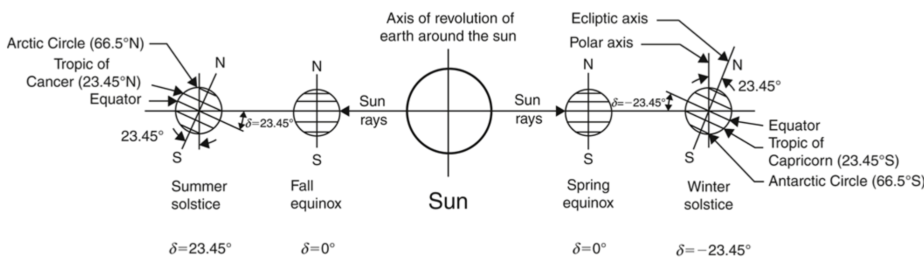




# Graph of declination angle



# Yearly variation of solar declination



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# *Hour Angle, h*

- The hour angle h, of a point on the earth's surface is defined as the angle through which the earth would turn to bring the meridian of the point directly under the sun.
- The hour angle at local solar noon is zero, with each 360/24 or 15 degrees of longitude equivalent to one hour, afternoon hours being designated as positive. Expressed symbolically, the hour angle in degrees is:

  **h = ±0.25 (number of minutes from local solar noon)**

  where the + sign applies to afternoon hours and the - sign to morning hours.
- The hour angle can also be obtained from the apparent solar time (AST), i.e., the corrected local solar time): **h = (AST-12)15**
- At local solar noon AST=12 and h=0°. Therefore, the Local Standard Time (the time shown by our clocks at local solar noon ) is:

  LST = 12 - ET ± 4 (SL-LL) [note: -/+]
- For the city in Germany: LST = 12 - ET - 9.2 [min]

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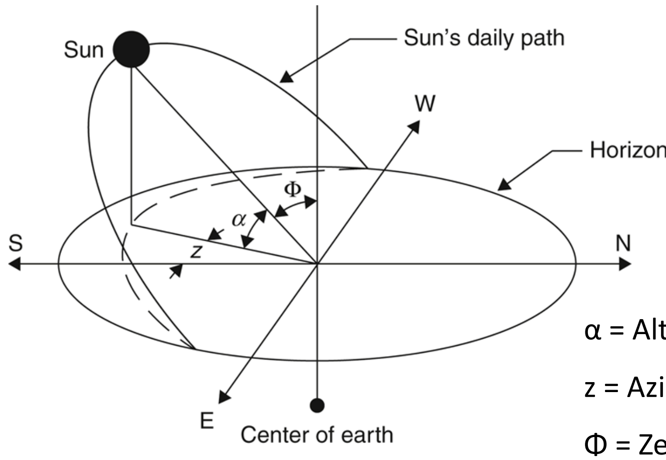
# Position of the sun in the sky




α = Altitude angle

z = Azimuth angle

Φ = Zenith angle





# *Solar Altitude Angle, α*

- The solar altitude angle is the angle between the sun's rays and a horizontal plane as shown in previous figure.
- It is related to the Solar Zenith angle Φ, being the complimentary angle between the sun's rays and the vertical.
- Thus:

$$\Phi+\alpha = \pi/2 = 90^{\circ}$$

- The mathematical expression for the solar altitude angle is:

$$\sin(\alpha) = \cos(\Phi) = \sin(L)\,\sin(\delta)+\cos(L)\,\cos(\delta)\,\cos(h)$$

where L = local latitude.

# *Solar Azimuth Angle, z*

- The solar azimuth angle z is the angle of the sun's rays measured in the horizontal plane from due south (true south); westward is designated as positive for the northern hemisphere.
- The mathematical expression for the solar azimuth angle is:

  **$\sin(z) = \cos(\delta) \sin(h)/\cos(\alpha)$**
- This equation is correct provided that:

  $\cos(h) > \tan(\delta) / \tan(L)$
- If not it means that the sun is behind the E-W line and the azimuth angle for the morning hours is $-\pi+|z|$ and for the afternoon hours is $\pi-z$.
- At solar noon, the sun is, by definition, exactly on the meridian, which contains the north-south line, and consequently, the solar azimuth is zero degrees.
- Therefore the noon altitude $\alpha_n$ is: **$\alpha_n = 90^o - L + \delta$**





# *Sun Rise and Set Times and Day Length*

- The sun is said to rise and set when the solar altitude angle is zero. So the hour angle at sunset, $h_{ss}$, can be found from solving the equation for the altitude angle for h when $\alpha=0^o$. Thus :

  $\sin(\alpha) = \sin(0) = 0 = \sin(L)*\sin(\delta)+\cos(L)*\cos(\delta)*\cos(h_{ss})$

  which reduces to: **$\cos(h_{ss}) = -\tan(L) \tan(\delta)$**

  where $h_{ss}$ is taken as positive at sunset.
- Since the hour angle at local solar noon is zero, with each 15 degrees of longitude equivalent to one hour, the sunrise and sunset time in hours from local solar noon is then:

  **$H_{ss} = -H_{sr} = 1/15 \cos^{-1}[-\tan(L) \tan(\delta)]$**
- The local standard time at sunset for the city of Germany is:

  **Sunset Standard Time = $H_{ss}$-ET-9.2 (min)**
- The day length is twice the sunset hour since the solar noon is at the middle of the sunrise and sunset hours. Thus the length of the day in hours is :

  **Day Length = $2/15 \cos^{-1}[-\tan(L) \tan(\delta)]$**

# *Incidence Angle, Θ*

- The solar incidence angle, θ is the angle between the sun's rays and the normal on a surface.
- For a horizontal plane the incidence angle, θ and the zenith angle Φ are the same.
- The angles shown in previous figure are related to the basic solar angles with the following general expression for the angle of incidence:

  $\cos(\theta) = \sin(L)\sin(\delta)\cos(\beta) - \cos(L)\sin(\delta)\sin(\beta)\cos(Zs) + \cos(L)\cos(\delta)\cos(h)\cos(\beta) + \sin(L)\cos(\delta)\cos(h)\sin(\beta)\cos(Zs) + \cos(\delta)\sin(h)\sin(\beta)\sin(Zs)$

where:

β = surface tilt angle from the horizontal.

Zs = surface azimuth angle; angle between the normal to the surface from true south.

- For certain cases above equation reduces to much simpler forms.



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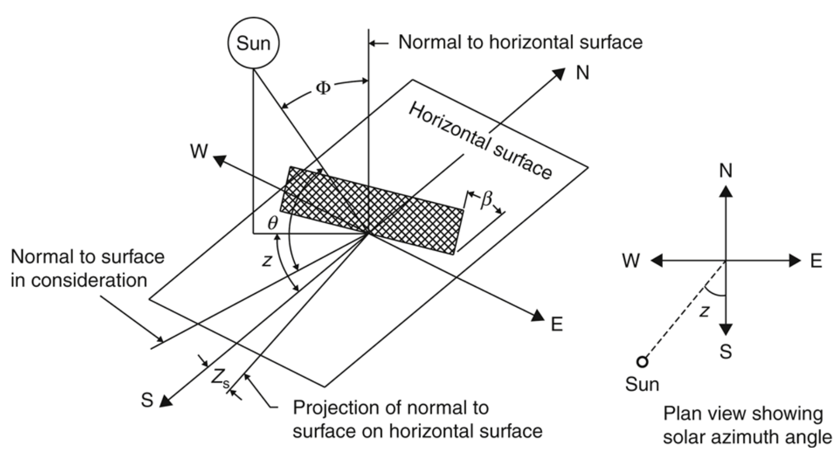
## Solar angles diagram for an inclined surface

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# Simplification of the general equation

- For horizontal surfaces: $\beta=0^o$ and $\theta=\Phi$, therefore general equation reduces to equation for $\alpha$.
- For vertical surfaces: $\beta=90^o$ and general equation becomes:
  $\cos(\theta)=-\cos(L)\sin(\delta)\cos(Zs)+\sin(L)\cos(\delta)\cos(h)\cos(Zs)+\cos(\delta)\sin(h)\sin(Zs)$
- For south facing tilted surface in the northern hemisphere: $Zs=0^o$ and general equation reduces to:
  $\cos(\theta)=\sin(L)\sin(\delta)\cos(\beta)-\cos(L)\sin(\delta)\sin(\beta)+\cos(L)\cos(\delta)\cos(h)\cos(\beta)+\sin(L)\cos(\delta)\cos(h)\sin(\beta)$
  which can be further reduced to:
  $$\cos(\theta)=\sin(L-\beta)\sin(\delta)+\cos(L-\beta)\cos(\delta)\cos(h)$$
- For a north facing tilted surface in the southern hemisphere: $Zs=180^o$ and general equation reduces to:
  $$\cos(\theta)=\sin(L+\beta)\sin(\delta)+\cos(L+\beta)\cos(\delta)\cos(h)$$



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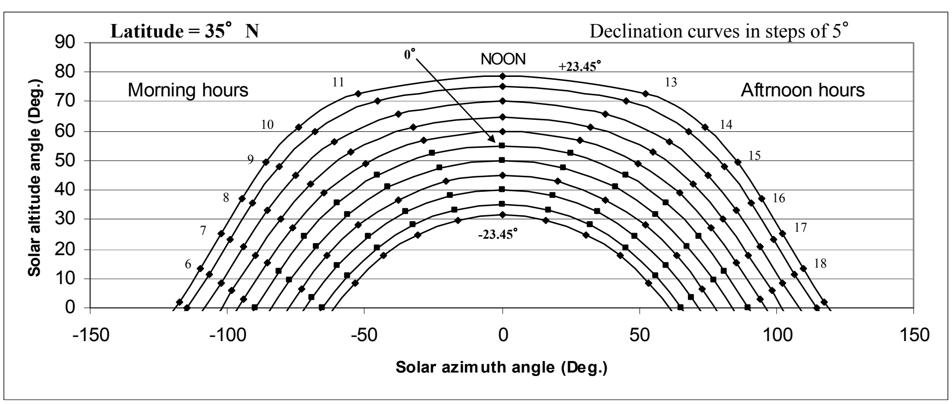
# Solar Diagrams-Cylindrical

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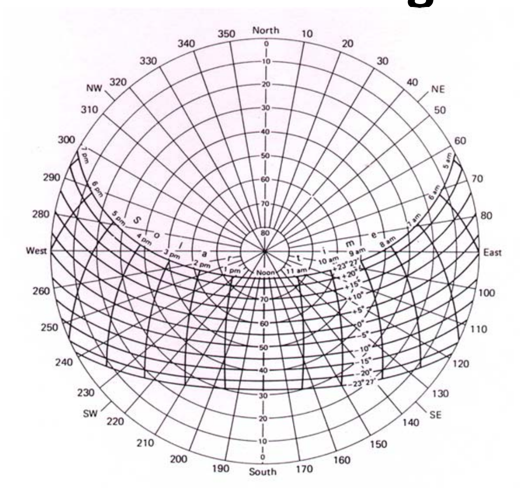
# Solar Diagrams-Polar




| Declination | Approximate dates |
|---|---|
| +23° 27′ | June 22 |
| +20° | May 21, July 24 |
| +15° | May 1, August 12 |
| +10° | April 16, August 28 |
| +5° | April 3, September 10 |
| 0° | March 21, September 23 |
| -5° | March 8, October 6 |
| -10° | February 23, October 20 |
| -15° | February 9, November 3 |
| -20° | January 21, November 22 |
| -23° 27′ | December 22 |



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# Solar radiation

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# SOLAR RADIATION

- All substances, solid bodies as well as liquids and gases above the absolute zero temperature, emit energy in the form of electromagnetic waves.
- The radiation which is important to us is that emitted by the sun which lies within the ultraviolet, visible and infrared regions.
- The radiation wavelength which is important to solar energy application is between 0.15 and 3.0 μm.
- The wavelengths in the visible region lie between 0.38 and 0.72 μm.



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# Solar constant

- The amount of solar energy received per unit time per unit area at the mean distance of the earth from the sun on a surface normal to the sun is called the solar constant $G_{sc}$.
- This quantity is difficult to measure from the surface of the earth because of the effect of the atmosphere.
- When the sun is closest to the earth, on December 21 the solar heat on the outer edge of the earth's atmosphere is about 1400 W/m$^2$ and when the sun is farthest away on June 21 is about 1310 W/m$^2$.

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# Variation of extraterrestrial radiation

- Throughout the year the solar heat $G_{on}$ varies between these limits as indicated in next figure in the range of 3% and can be calculated by:

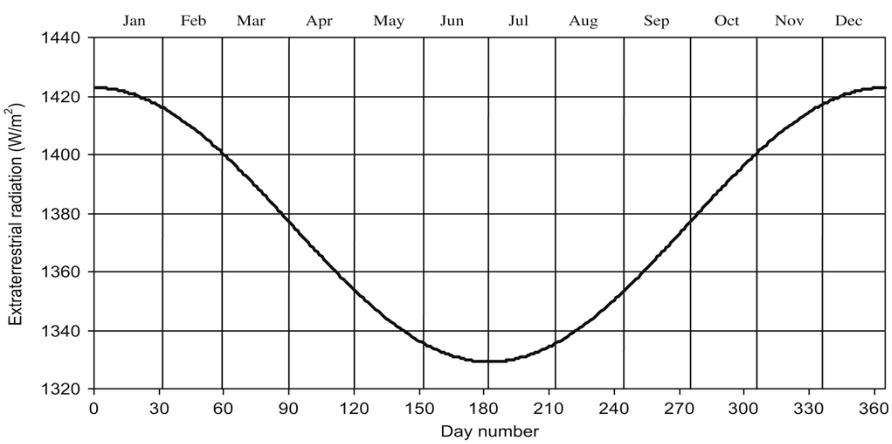
$$G_{on} = G_{sc} [ 1+0.033 \cos(360 N / 365)]$$

- Where: $G_{on}$ = extraterrestrial radiation measured on the plane normal to the radiation on the Nth day of the year ($W/m^2$); and
- $G_{sc}$ = solar constant, ($W/m^2$).
- The latest value of $G_{sc}$ as obtained from recent satellite data and adopted by the World Radiation Center, is 1367 $W/m^2$ with an estimated error of 1%.

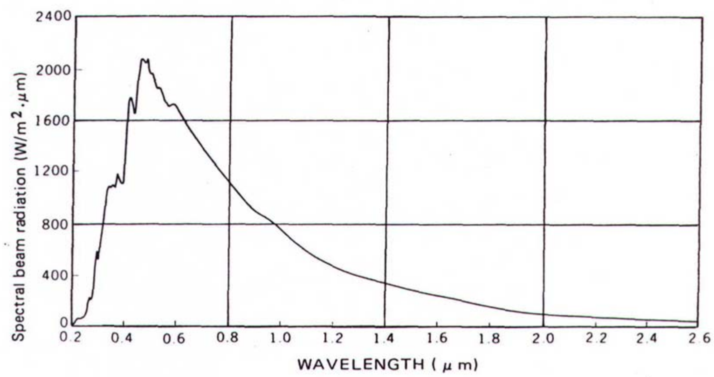




# Variation of extraterrestrial solar radiation



# Standard spectrum giving a solar constant of 1367 W/m²



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# Extraterrestrial radiation on a horizontal plane

- Extraterrestrial radiation on a horizontal plane for day N is given by:

$$G_{oH} = G_{on}\cos(\Phi) = G_{sc}\left[1 + 0.033\cos\left(\frac{360N}{365}\right)\right]\left(\cos(L)\cos(\delta)\cos(h)+\sin(L)\sin(\delta)\right)$$

- For the whole day by integrating the above relation from sunrise to sunset:

$$H_o = \frac{24x3600G_{sc}}{\pi}\left[1 + 0.033\cos\left(\frac{360N}{365}\right)\right]\left(\cos(L)\cos(\delta)\sin(h_{ss}) + \left(\frac{\pi h_{ss}}{180}\right)\sin(L)\sin(\delta)\right)$$

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# Solar radiation on earth's surface

- The solar heat reaching the Earth's surface is reduced below $G_{on}$ because a large part of it is scattered, reflected back out into space and absorbed by the atmosphere.
- The scattered radiation is termed diffuse or sky radiation.
- The solar heat that comes directly through the atmosphere is termed direct or beam radiation.
- The solar heat at any point on earth depends on:

  1. The ozone layer thickness.
  2. The distance traveled through the atmosphere to reach that point.
  3. The amount of haze in air (dust particles, water vapor, etc.).
  4. The extent of the cloud cover.

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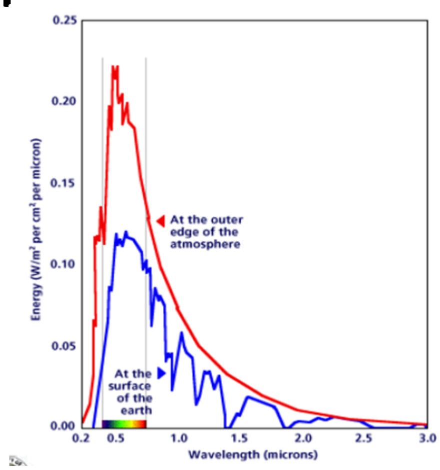
# Solar spectrum at earth's surface

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# Air mass

- The earth is surrounded by an atmosphere which contains various gaseous constituents, suspended dust and other minute solid and liquid particulate matter and clouds of various types.
- Therefore, the solar radiation is depleted during its passage though the atmosphere before reaching the earth's surface.
- The reduction of intensity with increasing zenith angle of the sun is generally assumed to be directly proportional to the increase in air mass, an assumption that considers the atmosphere to be unstratified with regard to absorbing or scattering impurities.
- The degree of attenuation of solar radiation traveling through the earth's atmosphere is dependent on the length of path and the characteristics of the medium traversed.
- In solar radiation calculations, one standard ***air mass*** is defined as the length of path traversed in reaching the sea level when the sun is at the zenith (the vertical at the point of observation).



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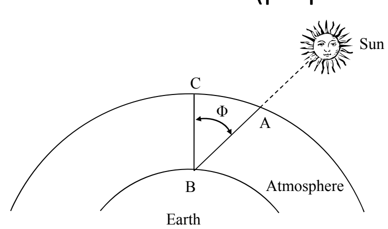
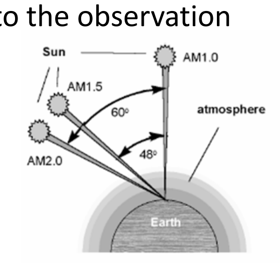
# Air mass

- In solar energy calculations air mass is defined as the length of corridor that the radiation needs to pass to reach sea-level when the sun is at its zenith (perpendicular to the observation point).




- The air mass is related to the zenith angle Φ, without considering the earth's curvature, by the equation:

$$m = \frac{AB}{BC} = \frac{1}{\cos(\Phi)}$$

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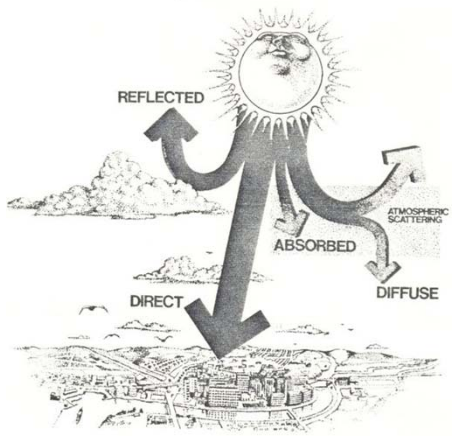
# Available solar radiation

- Direct, beam radiation
- Diffuse radiation
- Reflected radiation



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# Symbols

- G-irradiance ($W/m^2$)
- Insolation or irradiation (integration of solar irradiance over a time frame), ($J/m^2$). Thus we have:
  - H = for a day
  - I = for an hour
- Subscripts
  - B = Beam (direct)
  - D = Diffuse
  - G = Ground reflected

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# Terrestrial Irradiation

- The relation between the diffuse and total radiation based on the monthly average clearness index $\overline{K}_T$ is (empirical relation):

$$\frac{\overline{H}_D}{\overline{H}} = 1.390 - 4.027\overline{K}_T + 5.531\overline{K}_T^2 - 3.108\overline{K}_T^3$$

- Where: $\overline{K}_T = \frac{\overline{H}}{\overline{H}_o}$
- $\overline{H}$ monthly average total insolation on a terrestrial horizontal surface (MJ/m$^2$-day) [from tables]
- $\overline{H}_o$ monthly average daily total insolation on an extraterrestrial horizontal surface (MJ/m$^2$) – estimated from the relation we have seen already or from tables for a particular day of the month where solar radiation is equal to the mean monthly value.



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**Monthly average daily extraterrestrial insolation on horizontal surface (MJ/m$^2$)**

| Latitude | Jan 17 | Feb 16 | Mar 16 | Apr 15 | May 15 | June 11 | July 17 | Aug 16 | Sept 15 | Oct 15 | Nov 14 | Dec 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 60°S | 41.1 | 31.9 | 21.2 | 10.9 | 4.4 | 2.1 | 3.1 | 7.8 | 16.7 | 28.1 | 38.4 | 43.6 |
| 55°S | 41.7 | 33.7 | 23.8 | 13.8 | 7.1 | 4.5 | 5.6 | 10.7 | 19.5 | 30.2 | 39.4 | 43.9 |
| 50°S | 42.4 | 35.3 | 26.3 | 16.8 | 10.0 | 7.2 | 8.4 | 13.6 | 22.2 | 32.1 | 40.3 | 44.2 |
| 45°S | 42.9 | 36.8 | 28.6 | 19.6 | 12.9 | 10.0 | 11.2 | 16.5 | 24.7 | 33.8 | 41.1 | 44.4 |
| 40°S | 43.1 | 37.9 | 30.7 | 22.3 | 15.8 | 12.9 | 14.1 | 19.3 | 27.1 | 35.3 | 41.6 | 44.4 |
| 35°S | 43.2 | 38.8 | 32.5 | 24.8 | 18.6 | 15.8 | 17.0 | 22.0 | 29.2 | 36.5 | 41.9 | 44.2 |
| 30°S | 43.0 | 39.5 | 34.1 | 27.2 | 21.4 | 18.7 | 19.8 | 24.5 | 31.1 | 37.5 | 41.9 | 43.7 |
| 25°S | 42.5 | 39.9 | 35.4 | 29.4 | 24.1 | 21.5 | 22.5 | 26.9 | 32.8 | 38.1 | 41.6 | 43.0 |
| 20°S | 41.8 | 39.9 | 36.5 | 31.3 | 26.6 | 24.2 | 25.1 | 29.1 | 34.2 | 38.5 | 41.1 | 42.0 |
| 15°S | 40.8 | 39.7 | 37.2 | 33.1 | 28.9 | 26.8 | 27.6 | 31.1 | 35.4 | 38.7 | 40.3 | 40.8 |
| 10°S | 39.5 | 39.3 | 37.7 | 34.6 | 31.1 | 29.2 | 29.9 | 32.8 | 36.3 | 38.5 | 39.3 | 39.3 |
| 5°S | 38.0 | 38.5 | 38.0 | 35.8 | 33.0 | 31.4 | 32.0 | 34.4 | 36.9 | 38.1 | 37.9 | 37.6 |
| 0 | 36.2 | 37.4 | 37.9 | 36.8 | 34.8 | 33.5 | 33.9 | 35.7 | 37.2 | 37.3 | 36.4 | 35.6 |
| 5°N | 34.2 | 36.1 | 37.5 | 37.5 | 36.3 | 35.3 | 35.6 | 36.7 | 37.3 | 36.3 | 34.5 | 33.5 |
| 10°N | 32.0 | 34.6 | 36.9 | 37.9 | 37.5 | 37.0 | 37.1 | 37.5 | 37.0 | 35.1 | 32.5 | 31.1 |
| 15°N | 29.5 | 32.7 | 35.9 | 38.0 | 38.5 | 38.4 | 38.3 | 38.0 | 36.5 | 33.5 | 30.2 | 28.5 |
| 20°N | 26.9 | 30.7 | 34.7 | 37.9 | 39.3 | 39.5 | 39.3 | 38.2 | 35.7 | 31.8 | 27.7 | 25.7 |
| 25°N | 24.1 | 28.4 | 33.3 | 37.5 | 39.8 | 40.4 | 40.0 | 38.2 | 34.7 | 29.8 | 25.1 | 22.9 |
| 30°N | 21.3 | 26.0 | 31.6 | 36.8 | 40.0 | 41.1 | 40.4 | 37.9 | 33.4 | 27.5 | 22.3 | 19.9 |
| 35°N | 18.3 | 23.3 | 29.6 | 35.8 | 39.9 | 41.5 | 40.6 | 37.3 | 31.8 | 25.1 | 19.4 | 16.8 |
| 40°N | 15.2 | 20.5 | 27.4 | 34.6 | 39.7 | 41.7 | 40.6 | 36.5 | 30.0 | 22.5 | 16.4 | 13.7 |
| 45°N | 12.1 | 17.6 | 25.0 | 33.1 | 39.2 | 41.7 | 40.4 | 35.4 | 27.9 | 19.8 | 13.4 | 10.7 |
| 50°N | 9.1 | 14.6 | 22.5 | 31.4 | 38.4 | 41.5 | 40.0 | 34.1 | 25.7 | 16.9 | 10.4 | 7.7 |
| 55°N | 6.1 | 11.6 | 19.7 | 29.5 | 37.6 | 41.3 | 39.4 | 32.7 | 23.2 | 13.9 | 7.4 | 4.8 |
| 60°N | 3.4 | 8.5 | 16.8 | 27.4 | 36.6 | 41.0 | 38.8 | 31.0 | 20.6 | 10.9 | 4.5 | 2.3 |

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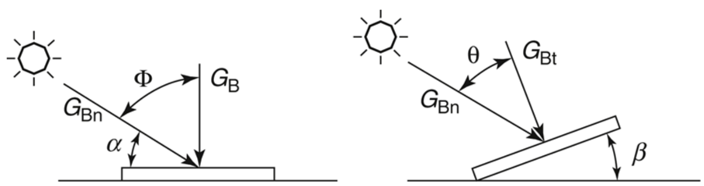
# Total Radiation on Tilted Surfaces

The amount of solar radiation on a surface on earth at any position at a particular time depends on the orientation and inclination of a surface.

An inclined surface takes direct ($G_{Bt}$), diffuse ($G_{Dt}$) and ground reflected ($G_{Gt}$) radiation given by:

$$G_t = G_{Bt} + G_{Dt} + G_{Gt}$$



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# Beam radiation tilt factor

- As shown from the previous diagrams:

  $G_{Bt}=G_{bn}\cos(\theta)$ and $G_B=G_{Bn}\cos(\Phi)$

- So the <u>beam radiation tilt factor</u>: $R_B = \frac{G_{Bt}}{G_B} = \frac{\cos(\theta)}{\cos(\Phi)}$
- So for a stationary surface facing south with inclination β:

$$R_B = \frac{\cos(\theta)}{\cos(\Phi)} = \frac{\sin(L-\beta)\sin(\delta)+\cos(L-\beta)\cos(\delta)\cos(h)}{\sin(L)\sin(\delta)+\cos(L)\cos(\delta)\cos(h)}$$

- So the direct radiation for any surface is given by:

$$G_{Bt} = G_B R_B$$

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# Total radiation on a tilted surface

- Using the isotropic sky model:

$$G_t = R_B G_B + G_D\left(\frac{1+\cos(\beta)}{2}\right) + (G_B + G_D)\rho_G\left(\frac{1-\cos(\beta)}{2}\right)$$

- The total radiation on a flat surface G, is the sum of horizontal bean and diffuse radiation : $G = G_B + G_D$
- Therefore we can find a new factor called total radiation tilt factor:

$$R = \frac{G_t}{G} = \frac{G_B}{G}R_B + \frac{G_D}{G}\left(\frac{1+\cos(\beta)}{2}\right) + \rho_G\left(\frac{1-\cos(\beta)}{2}\right)$$



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- Knowing the average total daily radiation for the month and the corresponding diffuse radiation, the average beam radiation on a horizontal surface can be obtained from: $\overline{H}_B = \overline{H} - \overline{H}_D$
- As before the monthly total radiation tilt factor is:

$$\overline{R} = \frac{\overline{H}_t}{\overline{H}} = \left(1 - \frac{\overline{H}_D}{\overline{H}}\right)\overline{R}_B + \frac{\overline{H}_D}{\overline{H}}\left(\frac{1+\cos(\beta)}{2}\right) + \rho_G\left(\frac{1-\cos(\beta)}{2}\right)$$

- Where $\overline{H}_t$ = monthly average daily total radiation on a tilted surface and $\overline{R}_B$ = monthly mean radiation beam factor

(2.109)

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# Monthly mean radiation beam factor

- For surfaces facing the equator:

$$\overline{R}_B = \frac{\cos(L-\beta)\cos(\delta)\sin(h'_{ss}) + (\pi/180)h'_{ss}\sin(L-\beta)\sin(\delta)}{\cos(L)\cos(\delta)\sin(h_{ss}) + (\pi/180)h_{ss}\sin(L)\sin(\delta)}$$

- And the sunset hour angle on the tilted surface is:

$$h'_{ss} = \min\{h_{ss}, \cos^{-1}[-\tan(L-\beta)\tan(\delta)]\}$$



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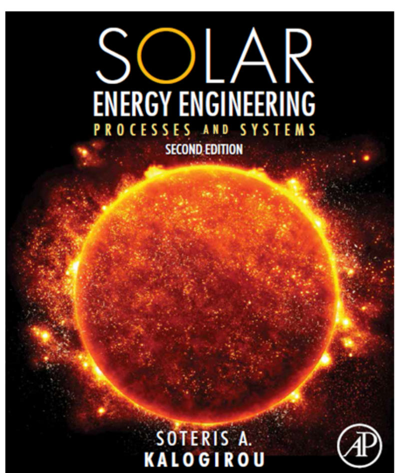
## This and other information can be found in….

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## Thank you for your attention,

any questions please….